Teaching philosophy and career goals

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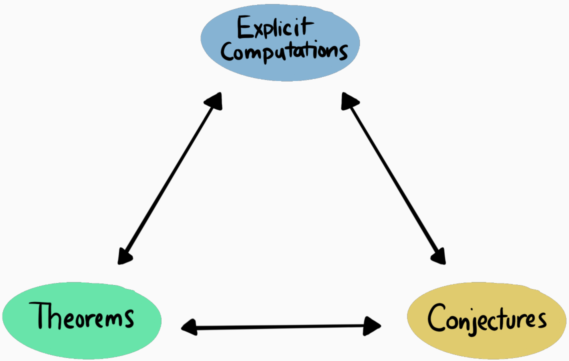
- and other people

Explicit computations



I develop and implement robust algorithms for arithmetic geometry.

By making them accessible and easy to use by others, I have enabled many others to **explore** mathematics via **experimentation**.

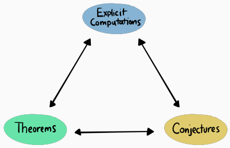
Computational arithmetic geometry

Arithmetic geometry = algebraic geometry + number theory

For example, studying integer/rational solutions of a system of polynomials.

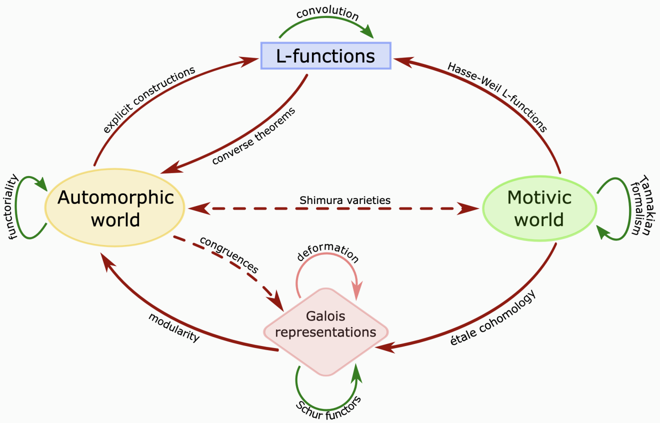
- Curves: theory and computation have been developed in synchrony
- Surfaces and beyond: theory has advanced way beyond computation
  - Combinatorial explosion deters initial computations
  - Faster and big computers are not enough
  - We need efficient algorithms and insights

My research program is focused on enabling the full triangle



I build and use the infrastructure to understand patterns in arithmetic geometry.

The motivation: Langlands' program



I provide compelling visual and computational displays for the program through the L-functions and Modular Forms Database https://www.lmfdb.org/

Plenty left to understand theoretically and explicitly. Many student projects.

Teaching

- I have taught at various places: Portugal, NYU, Dartmouth, MIT
- In a large range of settings:
    - Large classes: Complex analysis and differential equations: 200+ students
    - Introductory level: Calculus I: 90+ students
    - Mid level: Probability, Differential equations, Number theory
    - Upper level: Mathematical Cryptography, Introduction to Algebraic Geometry
- My goals as a teacher are:
    - To assist students in learning
    - To expose students to new ways of thinking
- My most successful tools have been:
    - Communication
    - Adaptation

Communication

Setting expectations is key for a successful class

Here are some things that worked out for me so far:

- Seek feedback
  - Send anonymous surveys
  - Build dialogue with the students outside classroom
- Provide active feedback
- Build a highly participatory classroom
  - Invite students to explore with me
  - It is okay to say “I do not know” or simply be wrong
  - Provide tools for students to learn from their mistakes

Active feedback

One of my major goals is to help students learn.

Students learn with a lot of practice, but practice without feedback is not helpful!

Here are some ideas that I have:

- Web homework
- Khan academy
- Encourage the use of computer algebra systems
- Lean - Interactive proof checker
- 2×2 Rubik's cube
- Encourage collaboration
- Devise grading strategies that allow students to learn from their mistakes
    - Enable them to resubmit homework

Developing critical thinking

My other major goal is to expose students to new ways of thinking.

I try to break concepts into two steps

- Big picture
- Formalization

Recall: Theorem 3.2.3

Theorem 3.2.3

Let $y_1(t)$ and $y_2(t)$ be two **solutions** of the homogeneous ODE

$$y'' + p(t)y' + q(t)y = 0,$$

that are defined at t_0.

If $W(y_1, y_2)(t_0) \neq 0$, then **every** solution to the initial value problem

$$y(t_0) = y_0, \quad y'(t_0) = y_0'$$

can solved in the form

$$y(t) = C_1 y_1(t) + C_2 y_2(t).$$

What is the meaning of $W(y_1, y_2)(t_0) \neq 0$? What is the key idea?

Fundamental Solution set

Definition

Assume that $p(t)$ and $q(t)$ are **continuous** in (α, β).
Further, assume that $y_1(t)$ and $y_2(t)$ are two solutions in (α, β) for

$$y'' + p(t)y' + q(t)y = 0.$$

If $W(y_1, y_2)(t_0) \neq 0$ for **some** $t_0 \in (\alpha, \beta)$ then $\{y_1(t), y_2(t)\}$ is called the **fundamental solution set** of the ODE on the interval (α, β).

Q: Why do we call them **fundamental**?
A: Because every solution in (α, β) can be written as $y(t) = C_1y_1(t) + C_2y_2(t)$
Even if the initial conditions are given at $t_1 \neq t_0$, with $t_1 \in (\alpha, \beta)$

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I encourage my students to be curious and skeptical.

- What are the implications?
- What makes it all work?
- Where are we headed?

Many times, the path to the right answer is more important than the answer itself.